THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS
MATH2010D Advanced Calculus 2019-2020

Problem Set 3

1. Draw the following subsets of $\mathbb{R}^2$.

   (a) $D = \{(x, y) : 0 \leq x \leq y\}$;

   (b) $D = \{(x, y) : x - y > 0\}$;

   (c) $D = \{(x, y) : xy \geq 0\}$;

   (d) $D = \{(x, y) : |x| + |y| < 1\}$.

   (Hint: Write down the equation $|x| + |y| = 1$ explicitly in every quadrant.)

2. Describe the following subsets of $\mathbb{R}^2$.

   (a) $D = \{(r, \theta) : 1 < r < 2\}$;

   (b) $D = \{(r, \theta) : 0 \leq r \leq 3, 0 \leq \theta \leq \pi\}$.

3. Match the following polar equations and curves.

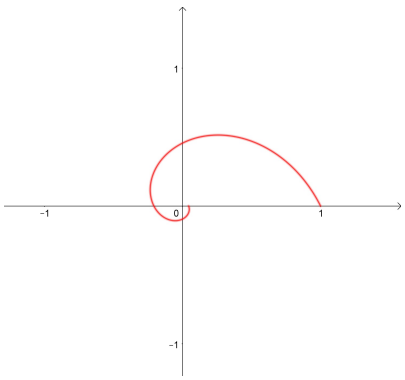
   (a) $r = \cos 2\theta$ for $0 \leq \theta \leq 2\pi$;

   (b) $r = \sin 2\theta$ for $0 \leq \theta \leq 2\pi$;

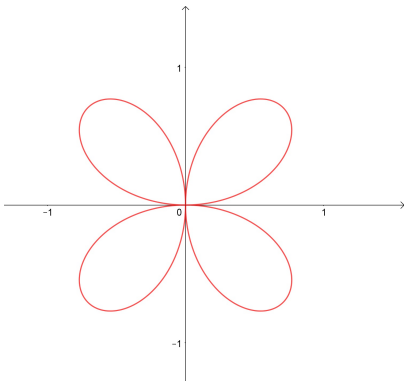
   (c) $r = e^{-\theta/2}$ for $0 \leq \theta \leq 2\pi$;

   (d) $r = \dfrac{1 - \cos\theta}{2}$ for $0 \leq \theta \leq 2\pi$.

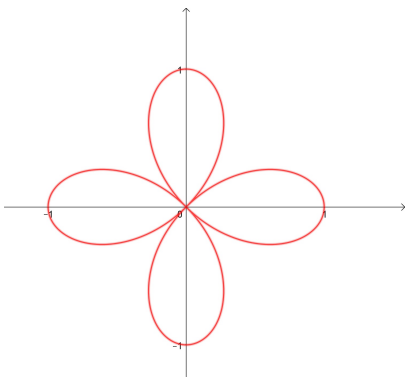
(i)



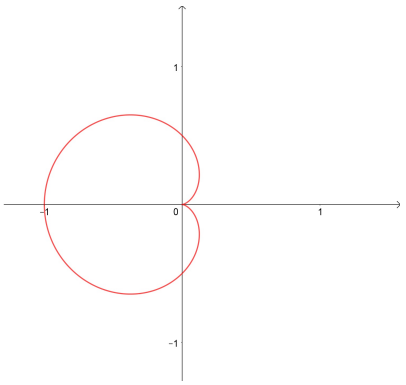
(iii)



(ii)



(iv)



4. Let $S = \{(x, 0) \in \mathbb{R}^2 \, : \, x \in \mathbb{R}\}$. Show that

   (a) $\text{Int}(S) = \phi$;

   (b) $\partial S = S$;

   (c) $\text{Ext}(S) = \{(x, y) \in \mathbb{R}^2 \, : \, x \in \mathbb{R}, \, y \neq 0\}$.

5. Let $S = \{\dfrac{1}{n} \, : \, n \in \mathbb{Z}^+\}$ be a subset of $\mathbb{R}$.

   Write down $\text{Int}(S)$ and ∂S.

6. Let $S = \{(x, y) \in \mathbb{R}^2 : |x| \geq 1\}$ be a subset of $\mathbb{R}^2$.

   Show that S is not path connected.

7. Let $S = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$ be a subset of $\mathbb{R}^2$.

   Show that S is a compact set.

8. Let $S = \{(e^t \cos t, e^t \sin t) \in \mathbb{R}^2 : t \in \mathbb{R}\}$ be a subset of $\mathbb{R}^2$. Prove that

   (a) S is unbounded;

   (b) $\mathbf{0} = (0, 0)$ is a boundary point of S.